BUTTERFLIES, BOXES and BINARIES

What a title for a chapter in an options book! What is their connection with options? Well, a butterfly is an options strategy – a combination, which should have placed it in chapter z. It’ll take some work, but you’ll see why it’s in this chapter. A binary is a type of option. It is, technically, an “exotic” option, because it’s not an “ordinary” call or put. We will see that it is, in fact, simpler than an ordinary option. And a box is a combination of options which produces a non-option outcome! These concepts don’t appear related but, as you will see, they are.

REVIEW: STRADDLES

We examines straddles in chapter z. Recall that a long straddle position is a purchase of both a call and a put on the same underlying asset, with the same strike price and expiration. Typically, the strike is at-(or near) the-money. Figure I shows a straddle’s payoff. The options are struck at 30, with the call and put premiums 1.75 and 1.25, respectively. Any movement in the underlying’s price away from the strike will place either the put or the call in-the-money, so the payoff is positive since the out-of-the-money option will simply expire. Will the net profit be positive? Only if the payoff exceeds the cost of the position – the sum of the two premiums. As Figure II shows, the net profit will exceed zero if the spot price of the underlying passes either of the breakeven points at expiration – 27 or 33. A straddle buyer, therefore, has no view as to the direction of the underlying. (Otherwise, she’d be paying for an option inconsistent with her view.) Her view is only that the price will change, and change enough to cross the breakeven point.

A seller (writer) of a straddle must have the opposite view – that the price will relatively stable. That is, stay within the two breakeven prices, so that he can keep at least part of the premiums received when the options were sold.

Exercise: Change in expiry.

BUTTERFLY

A standard butterfly comprises four option positions. JetBlue is 25/share. Let’s start by creating an “all-call” butterfly:

Position Premium

Write 22-Call 4.3

Buy 25-Call 2.5

Buy 25-Call 2.5

Write 28-Call 1.2

The 25-Calls are separated into two positions for reasons that will become clear in a later section. The options are European or American, and all expire in one year.[[1]](#footnote-1) The following table shows the payoffs for each option at various JetBlue prices and the total for the position. The Total column reflects the butterfly portfolio while each Call column refers to that particular option. For example, if JetBlue is 26 at expiration, the 22-Call requires a payment of 4 by the writer. But he also owns two 25-Calls, which pay 1 each, for a total payoff of −2.

TABLE I

BUTTERFLY PAYOFFS

Write 22-Call; Buy 2x25-Calls; Write 28-Call

JetBlue price

at expiration Call-22 Call-25 Call-28 Total Net

20 0 0 0 0 .5

21 0 0 0 0 .5

22 0 0 0 0 .5

23 1 0 0 -1 -.5

24 2 0 0 -2 -1.5

25 3 0 0 -3 -2.5

26 4 1 0 -2 -1.5

27 5 2 0 -1 -.5

28 6 3 0 0 .5

29 7 4 1 0 .5

30 8 5 2 0 .5

Figure 3 plots the Total payoff. Notice the two flat sections at each end – the butterfly “wings” – and the drop-down in the middle – the butterfly “belly.” No one would enter this position without compensation, as the cash flows are either zero or negative. Obviously, the premiums need to be recognized. The net premium received is +4.3 −2x2.5 +1.2 = +0.5, which is recognized in the P/L column. Figure 4 shows the butterfly P/L. Two points regarding this butterfly warrant emphasis:[[2]](#footnote-2)

* An investor entering this position is clearly betting on volatility. JetBlue needs to drop below 22.5 or rise above 27.5 in order to produce a net profit.
* The investor *receives* 0.5 at inception. Yet, if nothing changes – that is, JetBlue’s spot price at the butterfly’s expiration is the same as at the trade initiation – the investor incurs a *net loss* of 2.5.

An investor with the opposite view – i.e., that JetBlue will be relatively stable by the expiration date – will do the reverse trade: buy the two wings and sell the belly. She *pays* the upfront net premium of 0.5., but she earns a net profit should JetBlue stay within the (22.5, 27.5) bounds, with profit increasing the closer the spot price at expiration to the current spot price of 25.

The similarly and contrast with a straddle is important. Consider an ATM straddle; i.e., centered on 25. The buyer believes JetBlue will be volatile enough to pass the breakeven points by expiration. However, unlike the butterfly, the long straddle’s profit is not capped.[[3]](#footnote-3) The potential loss, therefore, must be greater. We’ll come back to this near the end of the next section.

DUPLICATE BUTTERFLIES

Change the calls to puts in the butterfly above, keeping everything else intact. Observe something utterly remarkable: each row of the Total column for the Put-Butterfly matches up exactly with the corresponding (the “complement”) row for the Call-Butterfly! And that’s why the Net column is not included. Nor are the premiums of the puts presented. They are not necessary, in a sense. Because the Total column of the Put is a duplicate of that of the Call, the net premiums *must be equal*. Otherwise, a riskless profit is obtainable. Suppose the net premium of the Put-Butterfly is 1.5, compared to 0.5 for the Call-Butterfly above. A trader would enter the Call-Butterfly exactly as above, and do the reverse of this Put-Butterfly;[[4]](#footnote-4) that is, purchase the 22-Put, write two 25-Puts and purchase the 28-Put. The Totals exactly offset each other for any JetBlue price at expiration, and the trader walks away with a net profit of 1.5−0.5=1 no matter what happens in the future. As this opportunity would be available to anyone, the actions of market participants will force the premiums to adjust until the net premiums of the two butterflies are equal. The net column for Table II will then, of course, be the same as for Table I. This is an example of the results of arbitrage eliminating riskless profits.

TABLE II

COMPLEMENT BUTTERFLY PAYOFFS

Write 22-Put; Buy 2x25-Put; Write 28-Put

JetBlue price

at expiration Put-22 Put-25 Put-28 Total

20 2 5 8 0

21 1 4 7 0

22 0 3 6 0

23 0 2 5 -1

24 0 1 4 -2

25 0 0 3 -3

26 0 0 2 -2

27 0 0 1 -1

28 0 0 0 0

29 0 0 0 0

30 0 0 0 0

Let’s try this butterfly another way:

Write 22-Put; Buy 25-Put; Buy 25-Call; Write 28-Call

Once again, the premiums are omitted, for reasons we’ll get to later. Table III presents the individual and total payoffs, shown in Figure 5. This may be the most intuitive Butterfly. It is constructed by purchasing a 25-straddle, which forms the belly, and selling a 22,28-strangle. Unlike a simple straddle, the otherwise open-ended potential profits are capped by way of the options that are sold, which form the butterfly wings. The cost of the butterfly must, therefore, be less than that of the straddle. How much less? Get ready.

TABLE III

ALTERNATIVE BUTTERFLY PAYOFFS

Write 22-Put; Buy 25-Put; Buy 25-Call; Write 28-Call

JetBlue price

at expiration Put-22 Put-25 Call-25 Call-28 Total

20 2 5 0 0 3

21 1 4 0 0 3

22 0 3 0 0 3

23 0 2 0 0 2

24 0 1 0 0 1

25 0 0 0 0 0

26 0 0 1 0 1

27 0 0 2 0 2

28 0 0 3 0 3

29 0 0 4 1 3

30 0 0 5 2 3

Compare the Total column of this table to each of the previous. The first point to notice is the common pattern. In all three butterflies, as JetBlue’s price at expiration rises from 20 to 30, the total payoff is flat, then declines until it reaches its lowest point, then rises until it hits its original maximum, after which it flattens again. This, of course, is what a butterfly is all about. The second point is the difference in *level*, as opposed to the pattern. This last butterfly’s total payoff is higher by 3 at every price. Employing the same arbitrage argument as above, it must be the case that the total of the premiums for this butterfly exceeds that of the others by exactly 3. Hence, it must be the case that:

−22-Put +25-Put +25-Call −28-Call = −22-Call +2x25-Call −28-Call +3 = −22-Put +2x25-Put −28-Put +3

In other words, the premiums on these sets of options are not independently determined.[[5]](#footnote-5) We’ll return to this idea later in the chapter, and show explicitly the interdependence. For now we note that the total premium for this butterfly is a net cost of 2.5 (3 above the net *revenue* of 0.5 of the original butterfly). If JetBlue finishes where it began – its price at expiration equals 25 as it is now – the investor loses 2.5. She’s hoping for volatility – in particular, a breakout past 22.5 or 27.5. Note that in this case as well as in the two previous, the investor is “long” volatility. But here the investor experiences a negative cash flow at inception; with the earlier butterflies, there is an initial receipt of cash.

A butterfly constructed this way – purchasing a call and put with common (at-the-money) strike price and writing a corresponding pair of (out-of-the-money) options to reduce the total premium – is more intuitive than the butterflies we began with. We’ll stick with these types of butterflies for the remainder of this chapter. Look closely at this quartet of options. The investor has purchased a 25-straddle and written a [22,28]-strangle. This butterfly will serve as our base.

UNCOMMON BUTTERFLIES

The butterflies introduced above are common in the sense that:

* The center (belly) strike price is the spot price of the underlying.
* The wing strike prices are equidistant from the center.

Let’s change each of these.

Off-Center Butterfly

Consider the following butterfly:

Position Premium

Write 23-Put 1.7

Buy 24-Put 2.0

Buy 24-Call 3.0

Write 25-Call 2.5

If JetBlue is now 24/share, this butterfly would be similar to those above in the sense that it is centered around 24. We would refer to it as an at-the-money butterfly. But JetBlue’s spot price is not 24; it is 25. Indeed, should JetBlue’s price at the butterfly’s expiration be back at 25 the only option with a non-zero payoff is the 24-Call which is owned. As this produces a positive payoff, we can term the butterfly as in-the-money.

What is the investor’s view in this case? Should JetBlue increase from its current price, the payoff does not get any higher as the 25-Call becomes in-the-money and subtracts any increase produced by the 24-Call. On the other hand, should JetBlue decrease, the payoff declines. The investor clearly does not want volatility; i.e., she is short volatility.

EXERCISE

1. Create a table similar to those above for this ITM-butterfly just discussed. Graph the resulting total payoff and net P/L.
2. What are the breakeven points for this butterfly? What are the “breakpoints;” i.e., the net P/L for the three strikes.
3. Where does this ITM-butterfly breakeven with ATM-butterfly examined at the end of the previous section?

Broken Butterfly

As our final example before we make a fundamental change in the Butterfly structure, consider the following:

Position Premium

Write 22-Put 1.3

Buy 25-Put 2.5

Buy 25-Call 2.5

Write 27-Call 1.6

This is similar to the base case Butterfly, and differs only in that a 27-Call is sold in place of the 28-Call. What does this accomplish? First, it breaks the symmetry, hence the name of this butterfly. More important, although the investor in this butterfly is long volatility as is the base butterfly investor, the out-of-pocket cost is reduced from 2.5 to 2.1. The trade-off, of course, is the lower potential profit. The exercises examine this more at length.

EXERCISES

1. a) Produce the payoff table for this Butterfly and draw the graph. Notice the “broken” wing.

b) What is this investor’s view compared to the purchaser of the base Butterfly above?

1. Analyze the following Broken Butterfly:

Position Premium

Write 25-Put 2.5

Buy 27-Put 3.6

Buy 27-Call 1.6

Write 28-Call 1.2

Compare this to the previous non-ATM (off-center) Butterfly. Why is that the relevant comparison? Remember, the spot price of the underlying is 25. Compare the net costs, breakevens and maximum profits.

FLAT BUTTERFLY

Return to the straddle of the first section: long a call and put, exercise price of 30, call and put premium 1.75 and 1.25, respectively. Suppose the investor is reluctant to pay a total premium equal to 10% of the underlying’s spot price for the sake of taking a position on volatility. Raising the strike of the call and lowering the strike for the put will do the trick. For example, the premium on a 31−call is 1.25 and for a 29-put it is 1. A long position in these two options was described as a strangle in chapter z. The strangle is a less aggressive strategy than the straddle. Because the sum of the premiums is lower – the strangle requires less of an investment – should the volatility view not be realized, the investor stands to lose less than with the straddle. This is seen in Figure xx, which should be compared to Figure 2. The breakeven points for the strangle (26.75, 33.25) extend further away from the spot price. This means a larger change in the underlying is required before net profit is generated.[[6]](#footnote-6) And, once the breakeven point is pierced by the underlying at expiration, the net profit for the straddle exceeds that of the strangle.

We can transform this strangle into a butterfly simply by writing a 27-put (premium=0.4) and writing a 33-call (premium=0.65). What have we accomplished by doing so?

* The net premium is reduced. Hence, the breakeven points are nearer to the spot price of the asset.
* The buyer has given up potential increased profits for movements in the underlying below 27 and above 33; i.e., a less aggressive long volatility position.

The exercise examines this position in more detail.[[7]](#footnote-7)

EXERCISE

1. Produce the payoff table for this flat butterfly and craw the graph.
2. Show that the prior butterflies can all be thought of (and reproduced) as a combination of a straddle and a strangle. This flat butterfly, on the other hand, is a combination of two strangles.

A BOX TRADE

XRT, the exchange-traded fund for the retail sector, is at 95. Let’s build a position comprising a quartet of one-year European options:[[8]](#footnote-8)

* Purchase 95-Call; Write 95-Put
* Write 99-Call; Purchase 99-Put

Consider the first pair. Suppose XRT finishes below 95 at the option’s expiration. The put buyer will exercise and you will be forced to pay 95 and receive XRT. If XRT finishes above 95 at expiration, you will exercise and pay 95 in exchange for a share of XRT. Should XRT be exactly 95 in one year, purchase it in the spot market for 95. In short, the pair of contacts are effectively a forward contract – you have agreed to pay 95 for XRT, regardless of its price on that date.

Similar, but reverse, arguments apply to the second pair of options. You have effectively locked yourself in to sell XRT for a price of 99 in one year. Now step back and think about the implications of the total position: you have entered a forward to buy STX in one year at 95 and you have entered a forward contract to sell STX in one year for a price of 99.[[9]](#footnote-9) In sum, regardless of STX’s price in one year, the quartet of options will provide you a net payment of $4.[[10]](#footnote-10) As this is a riskless future cash flow, its price – equivalently, its cost – must equal $4 discounted by the 1-year riskless interest rate, r. Or:

95-Call − 95-Put − 99-Call + 99-Put = 4/(1+r)

We have the following implication: the prices of the four options cannot be independently set by the market. Prices of any three completely determine the price of the fourth.

Note the difference between this relationship and put-call parity of chapter xx. Put-call parity states that, given the underlying spot price and the option pair’s exercise price, the put and call premiums are not independent – knowing one gives you the other. But the relationship hinges on the spot price of the underlying. Not so with the box relationship – it is independent of the spot price of STX. In other words, the box is a “dollar” relationship, as opposed to an “asset” relationship.[[11]](#footnote-11)

Another way to see this is by looking at the box from another perspective. Given the prices of all four options, we can solve for the interest rate – the “implied” interest rate. In other words:

* Do the box trade – it will require a net expenditure, say 3.9, depending on the net premium
* At expiration you will receive 4
* You earn an interest rate of 2.56%

You have, in effect, “lent” to the options market at 2.56%.[[12]](#footnote-12)

Exercises

1. For the example in the text, assume the following premiums:

95-Call 2

95-Put 1.02

99-Call 0.10

1. What is the 99-Put premium?
2. If, instead, the 99-Put premium is known to be 3, what must be the 9-Call premium?
3. What must be the relationship among the following *six-month* options on JetBlue?

25-Call; 25-Put; 26-Call; 26-Put

Assume the relevant six-month interest rate is 2%.

BACK TO BUTTERFLIES

Let’s briefly re-examine the original butterfly of this chapter and one of its compliments. The original involved only calls, with the net premium equal to −22-Call +2x25-Call −28-Call. The third consisted of a straddle and a strangle, with net premium of −22-Put +25-Put +25-Call −28-Call. We showed that the Payoff of the latter exceeds that of the former by 3 *for any price* of JetBlue at expiration. The difference between the two net premiums must, therefore, equal the present discounted value of 3: [[13]](#footnote-13)

−22-Call +2x25-Call −28-Call = −22-Put +25-Put +25-Call −28-Call + 3/(1+r)

Cancelling and collecting like terms we get:

[22-Call − 22-Put] + [− 25-Call + 25-Put] = 3/(1+r)

↓ ↓

Buy at 22 in one year & Sell for 25 in one year = value of 3 in one year

EXERCISE

Consider the four options in the Flat Butterfly section above:

* Purchase 29-Put and 31-Call
* Write 27-Put and 33-Call

Show that the following set of options:

* Write 27-Call and Purchase 29-Call
* Purchase 31-Call and Write 33-Call

produces the same payoff except separated by a fixed amount. Using the arbitrage arguments developed in this chapter, show how a quartet of options are related – a box.

INTRODUCING BINARIES

BINARIES IN COMBINATIONS

THE BINARY BOX

BINARY BUTTERFLY

1. The butterfly can be American or European. The arbitrage and box relationships later in the chapter, however, apply only to European style. [↑](#footnote-ref-1)
2. These two points are not shared by all butterflies. As we will see in the following sections and in exercises, there are four possibilities:

   long volatility and positive (i.e., net cost) total premium;

   long volatility and negative (i.e., net revenue) total premium;

   short volatility and positive total premium;

   short volatility and negative total premium. [↑](#footnote-ref-2)
3. Except, of course, at JetBlue hitting 0. [↑](#footnote-ref-3)
4. In this case the “trader” would be labeled an “arbitrageur.” [↑](#footnote-ref-4)
5. Recognizing non-zero interest rates simply requires discounting the 3, or 3/(1+rxt). This is a key idea later. [↑](#footnote-ref-5)
6. Once you understand the determinants of an option’s premium (chapter fff) you will appreciate why the strangle’s breakeven points must always be further than those of the straddle. [↑](#footnote-ref-6)
7. This type of butterfly is also known as a “condor,” the reason for which is, thankfully, beyond the scope of this book. [↑](#footnote-ref-7)
8. The Butterflies also involved a quartet of options. But the structures and goals are totally different, as we shall see. [↑](#footnote-ref-8)
9. This is why the word “box” is applied. In the equity markets, it describes a purchase of a stock and a short sale of the same stock. [↑](#footnote-ref-9)
10. As you can readily determine, the same results occur if the options are cash settled. [↑](#footnote-ref-10)
11. Write out the P-C Parity equation for the 95-Call and 99-Put, and for the 99-Call and 99-Put. Subtract one from the other – which removes the underlying – and you have the box equation. [↑](#footnote-ref-11)
12. You might be tempted to conclude that you can “borrow” from the options market at an attractive rate by doing the reverse trade. Sadly, the bid-ask spread, particularly for longer dated options and for options away from the money, will greatly reduce the attractiveness. [↑](#footnote-ref-12)
13. We assumed zero interest rates in our initial examination of the butterflies. [↑](#footnote-ref-13)